2005/2006

1 a) (i) $dy/dx = \frac{y\sqrt{x}}{1+y^2} \rightarrow \int \frac{1+y^2}{y}\,dy = \int \sqrt{x}\,dx \;;\; \int \frac{1}{y} + y\,dy = \int x^{1/2}\,dx$

$$\ln y + \tfrac{1}{2}y^2 = \frac{2x^{3/2}}{3} + C \;,\; \text{when } x=0,\ y=1$$

$$\ln 1 + \tfrac{1}{2} = C \rightarrow C = \tfrac{1}{2}$$

$$\ln y + \tfrac{1}{2}y^2 = \frac{2}{3}x^{3/2} + \ln 1 + \tfrac{1}{2}$$

$$2\ln y + y^2 = \frac{4}{3}x^{3/2} + 1$$

$$\ln y^2 + y^2 = \tfrac{4}{3}x^{3/2} + 1$$

2005/2006

(i) $dy/dx + \frac{1}{x}y = 1 + \frac{\sin 3x}{x}$ c/f $dy/dx + P(x)y = Q(x)$

I = Integration Factor = $e^{\int P(x)\,dx} = e^{\ln x} = x$

$$\Rightarrow \frac{d}{dx}(xy) = x\left(1 + \frac{\sin 3x}{x}\right)$$

$$xy = \int x + \sin 3x\,dx$$

$$xy = \frac{x^2}{2} - \frac{\cos 3x}{3} + C$$

$$y = \frac{x}{2} - \frac{\cos 3x}{3x} + \frac{C}{x}$$

(b) i) $d^2y/dx^2 + 3\,dy/dx - 4y = 0$

Aux. Eqn is $m^2 + 3m - 4 = 0 \Rightarrow (m-1)(m+4) = 0$

$\alpha = 1,\ \beta = -4$

$$\underline{y = Ae^{x} + Be^{-4x}}$$

ii) $d^2y/dx^2 - 4\,dy/dx + 4y = 0 \Rightarrow m^2 - 4m + 4 = 0$

$(m-2)^2 = 0 \quad m = 2 = \alpha = \beta$

$$\underline{y = e^{2x}(Ax + B)}$$

(iii) $4\,d^2y/dx^2 + y = 0 \rightarrow 4m^2 + 1 = 0 \quad m = \sqrt{-1/4} = \pm\tfrac{1}{2}i$

$(p \pm iq) = (0 \pm \tfrac{1}{2}i)$

$$y = e^{0x}(A\cos\tfrac{1}{2}x + B\sin\tfrac{1}{2}x)$$

$$y = A\cos\tfrac{1}{2}x + B\sin\tfrac{1}{2}x$$